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ESSAY · TRANSLATION

The Venus Pentagram and the Fibonacci Sequence

on beauty and its mathematical context

German original: *Das Venuspentagramm und die Fibonacchi-Folge*

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THIS ESSAY IN OTHER LANGUAGES

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Part 1

BACKGROUND IN CULTURAL HISTORY

The theme of aesthetics and beauty has always been a subject of mathematicians' reflections, above all because among the Greeks, the Babylonians and the Aztecs alike, mathematics as the basic science was at the same time the starting point of all other sciences. As a practising mathematician, however, one was also a co-creator and architect of being, which implicitly presupposed philosophical and religious reflections.

Whoever wanted to be a mathematician had to possess a comprehensive education, was versed in many disciplines and oriented towards the whole, which is why buildings, churches and temples embodied through their architectural form, besides the measurable parameters, also relations of vibration, which in turn corresponded to archetypal contents of consciousness. The aesthetics of a building was never merely an end in itself but revealed the world view of the builder or of the culture that stood behind it.

The golden section was one such concept, whose proportion was regarded as particularly harmonious. Since its continued fraction representation consists of nothing but ones, this was interpreted symbolically as a staircase, a chain or an "umbilical cord" which connected God (the origin, the unity, number 1) with creation – the human being.

phi (Φ) =

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cdots}}}}$$

The heavenly radiance of the stars therefore lived on in the earthly works (temples, cathedrals ...) and, from a mathematical point of view, rested above all on the special properties of the Fibonacci sequence, which in its most familiar form appears as the series of numbers 1-1-2-3-5-8-13-21... and whose terms approach the relation of the golden section or the pure numerical value Phi = 1.61803398875.

Mathematically there is a second number which in its continued fraction representation has a one in the numerator and only one single number in the denominator – the √2+1 – and which connects us with the octagon, since the length of the diagonal in the octagon has this measure. The octagon is thus inseparably linked through the continued fraction structure with the golden section and the pentagram, since in the pentagram the diagonal-to-side relationship is defined by the numerical value Phi = 1.618.

The golden section, the pentagram and the octagon are numbers and proportions which in cultural history are frequently linked with the concepts "beauty", "star" and "fertility", which is easy to understand even today, since our culture, too, connects aesthetics with the heavenly sphere and with the feminine.

Consequently, Venus as a planet is also linked with this, for in Proto-Sanskrit the word *ash-ta1* means both ***star and 8*** (eight). Accordingly, ash-ta ta-ra – as the origin of the words Ishtar, Isis or Ashera or Astarte – means eight-rayed star. The Olmecs, Maya and Aztecs also named this correspondence, for instance in the Tzolkin, where the qualities integrity, harmony and model were assigned to tone eight, and the concepts art, beauty and elegance corresponded to the seal of the star.

THE PATHS OF VENUS IN THE SKY – ON THE NUMBERS 5 AND 8

As can be shown, Venus as a planet connects, through its periodic movements in the sky, the numbers 8 and 5, the octagon and the pentagram, in an unsurpassable way. The following Figure 1 shows us the position of Venus on a particular calendar day over a period of eight years in relation to the Earth. We can observe how the 8 positions of Venus form a perfect octagon in the sky:

Figure 1

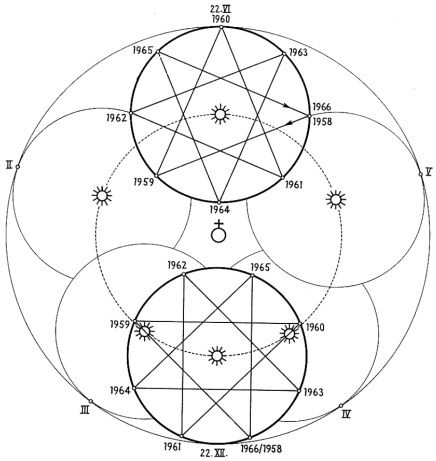


Fig. 110 Oktagramm der Venus-Stellungen 1958—1966, oben für das Datum des 22. Juni, unten für das des 22. Dezember.

This chart can be drawn up for any calendar date and forms an octagon, whereby the centre of the representation represents the Earth. If, in the course of a year, one picks out those octagon representations in which the points face the centre – astronomically speaking, Venus is in line with the Sun, defined astrologically as a conjunction – there are exactly 5 positions at intervals of ***73 calendar days or 72 degrees of angle*** where the rhythm of eight intersects with the rhythm of five, *see Figure 2*:

Figure 2

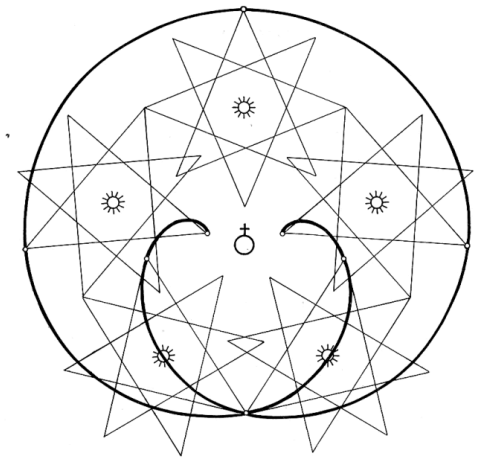
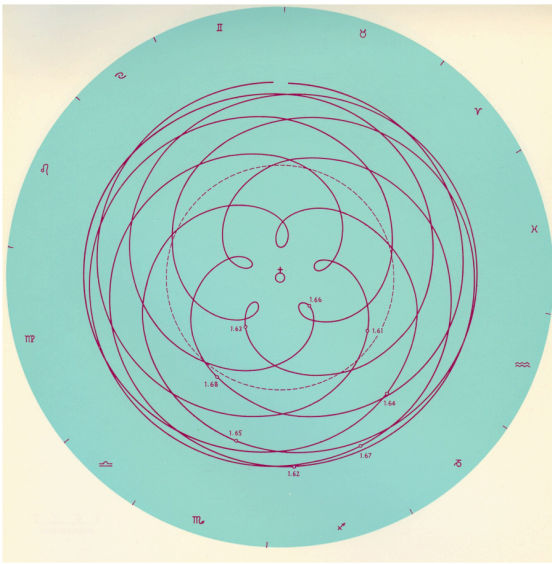


Fig. 111 Drehung des Stellungs-Oktagrammes der Venus während eines synodischen Umlaufes (1959—1961).

Figure 3 shows the geocentric orbit of Venus from the years 1960-1968, which forms the pentagram. The dashed line indicates the orbit of the Sun, whose radius is one astronomical unit.

Figure 3



Plates taken from the book: Rhythmen der Sterne, Joachim Schultz

The numbers 5 and 8 are linked with one another in many respects, as in the Fibonacci sequence, too, where they lie side by side as a pair of numbers:

(0) 1-1-2-3-**5-8**-13-21...

This Fibonacci sequence can be extended at will, whereby the approach to the limit Phi = (√5 +1)/2 = 1.61803398875 proceeds ever more exactly in alternation:

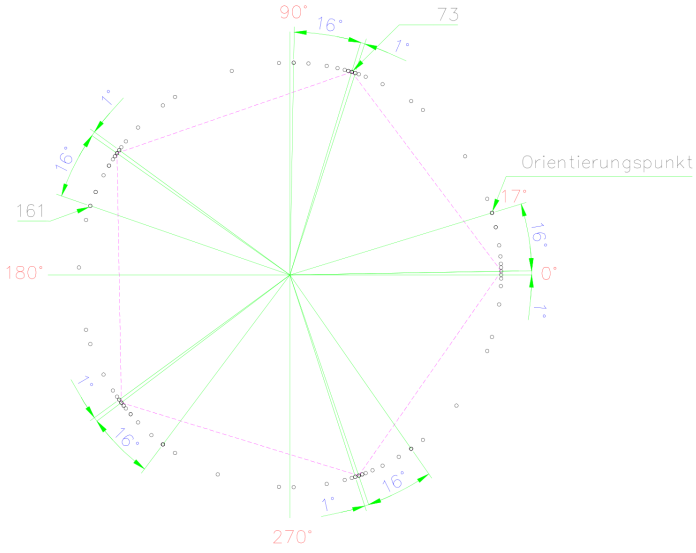
8 : 5 =	1.6	*less than phi*
13 : 8 =	1.625	greater than phi
21 : 13 =	1.61538...	*less than phi*
34 : 21 =	1.61904...	greater than phi
55 : 34 =	1.61764...	*less than phi*
89 : 55 =	1.61818...	greater than phi

If one goes on to regard this sequence of numbers as angle values or degrees on a circle, one recognises that exactly 13 numbers fit on the first circle (1,1,2,3,5,...144,233), and that in the transition from the 13th Fibonacci number (233) to the 14th Fibonacci number (377) the jump to the next circle takes place, since the value 377 corresponds exactly to 17° on the second circle: 360° + 17° = 377°. If one interprets the successive Phi values (ϕ) in this way and then maps them onto one single circle, 120 Fibonacci values form a perfect pentagram, see *Figure 4*.

Figure 4

Representation of 120 Fibonacci values in final circular projection

(the points correspond to the respective phi values)



The 73° as mean value and centre of the first ray of the pentagram is offset by 72° from the first degree of arc of the circle and illuminates the connection, shown in *Figure 2*, of the heavenly Venus pentagram, which is defined by the interval of 72 degrees of arc or 73 calendar days.

ϕ values:	Angle values in degrees of arc:	
1	1°	1st circle
1	1°	
2	2°	
3	3°	
5	5°	
8	8°	
13	13°	
21	21°	
34	34°	
55	55°	
89	89°	
144	144°	
233	233°	

377	17°	2nd circle	360° + 17°
610	250°	2nd circle	360° + 250°
987	267°	3rd circle	720° + 267°
1597	157°	4th circle	.....
2584	64°	7th circle	
4181	221°	11th circle	
6765	285°	18th circle	
10946	146°	.....	
17711	71°		
28657	217°		
46368	288°		
75025	145°		

.....

.....

.....

126493703204299739348832 2	2°	
204671111147398462369175 9	359°	
331164814351698201718008 1	1°	
535835925499096664087184 0	0°	
867000739850794865805192 1	1°	new beginning of the ϕ sequence
140283666534989152989237 61	1°	
.....	(2°)	
.....	(3°)	
.....	(5°)	
.....	(8°)	

If one examines the succession of the ϕ values in this way, one needs 120 terms of this sequence in order to build up the pentagram structure.
With the 121st Fibonacci value the above succession begins to repeat itself regularly.
We can note here that all values in degrees of arc can be arranged symmetrically about the centre value 161°, whereby the second numerical value 1° and the second-to-last numerical value 359° complement each other to 360°, the full circle. Accordingly, the fourth ϕ value 3° and the fourth-to-last 357° likewise add up to 360°, and so on.
The second significant symmetry which appears here is an exact mirroring of the original Fibonacci sequence within itself. As *Figure 5* shows, the Fibonacci sequence consists of two parts, whereby every second term of the sequence, as a reciprocal value, represents the difference to 360°. The 1 is thus allied with the 359 and vice versa, the 3 with the 357 and the 5 with the 355. So the 120th element of the Fibonacci series gives us the angle value 1°, the 118th element the value 2°, the 116th the value 5°, and so on.

Figure 5

120 Fibonacci values in degrees of arc, in sections of 60 terms each, in reduced (to 360°) form:

(0)	1	1	2	3	5	8	13	21	34	55	89	144
	161	161	322	123	85	208	293	141	74	215	289	144
233	17	250	267	157	64	221	285	146	71	217	288	
73	217	290	147	77	224	301	165	106	271	17	288	
145	73	218	291	149	80	229	309	178	127	305	72	
305	233	178	51	229	280	149	69	218	287	145	72	

17	89	106	195	301	136	77	213	290	143	73	216
217	289	146	75	221	296	157	93	250	343	233	216

289	145	74	219	293	152	85	237	322	199	161	
89	305	34	339	13	352	5	357	2	359	1	0
			21		8		3		1		
				13		5		2		1	

The total sum of the one hundred and twenty numerical values reduced to 360° comes to 19800, which can be broken down into the sums 8800 (the first five sections) and 11000 (the second five sections).

Representation of the sums of the ten sections of 12 elements each:

1 to 144: Σ 376 (232 + 144)	89 to 0: Σ 1856 (1712 + 144)
233 to 288: Σ 2216	217 to 216: Σ 2536
145 to 72: Σ 2176	305 to 72: Σ 2216
17 to 216: Σ 1856	73 to 288: Σ 2176
289 to 161: Σ 2176	161 to 144: Σ 2216

Σ 8800 Σ 11000 = Σ 19800

The value 19800 can also be broken down multiplicatively into the product 144 times 137.5, which corresponds exactly to the ϕ angle of the circle, since 360° divided by ϕ divides the circle into the angle sums 137.5° and 222.5°.

19800	=	144 × 137.5
360 / 1.61803398875…	=	222.49223595 ≈ 222.5
	→	137.5° + 222.5° = 360° full circle

Since some numbers occur twice in reduced form, we find among the first 120 elements 75 numbers that occur once and 45 that occur more than once. The relations of the numbers to one another likewise yield an image of the Fibonacci sequence:

Relation 120 : 75 : 45 = Relation 8 : 5 : 3

Let us now consider *Figure 6*, which orders the reduced Fibonacci values by size:

Figure 6

75 numerical values in degrees of arc ordered by size

359	71	143	215	287	
0 (360)	72	144	216	288	
1	73	145	217	289	"pack of five"
2	74	146	218	290	
3	75	147	219	291	
5	77	149	221	293	
8	80	152	224	296	
13	85	157	229	301	
17	89	161	233	305	
21	93	165	237	309	"pack of ten"
34	106	178	250	322	
51	123	195	267	339	
55	127	199	271	343	
64	136	208	280	352	
69	141	213	285	357	
				(359)	

The "packs of five"

are clearly distinguished from the "packs of ten" by the fact that the numerical values or degrees follow one another seamlessly as whole numbers and therefore always show a maximum difference of 4°: e.g.: 71° - 75°, 215° - 219°

The "packs of ten"

always differ from one another by a maximum difference of 64°. The 359 has a special position, since as a "bridging element" it can be assigned both to the beginning and to the end of the column of numbers. e.g.: 5° - 69°, 77° - 141°

The five "packs of five" give the sum.....	Σ 3985
The five "packs of ten" give the sum.....	Σ 8885
	Total sum 12870

The relationship of the total sums of the 120 and the 75 Fibonacci values gives the relation 19800 to 12870 or, simplified, 20 : 13. We find this correspondence again in the basic mathematical module of the "Maya Tzolkin", which is built up of 20 times 13 units.

The relation of the maximum differences of 64° and 4° corresponds to the relation 17 : 1.

The calendrical reference point of 17 January is of particular significance from a historical point of view, for in the Maya manner it symbolises the "rebirth and development of the new Earth" (number 1, sign of the dragon with the theme of birth and new beginning, and sign 17 – Earth – associated with synchronicity). In Western European development, too, 17 January plays an outstanding role, for it was the "favourite date" of the Prieuré de Sion.

The interfaces between the individual packs of numbers are fixed by the sequence 4/70-76/142-148/214-220/286-292/358 and are based on the development of the 72 (angle of the pentagram, 5 times 72° = 360°), whose multiples each represent the middle of the respective groups of numbers.

70 + 76 = 146,	146:2 =	middle	73 = 72 times 1 + 1
142 + 148 = 290,	290:2 =	middle	145 = 72 times 2 + 1
214 + 220 = 434,	434:2 =	middle	217 = 72 times 3 + 1

Representation of the midpoints of the pentagram:

17 = 0 + 17	1 = 0 + 1
89 = 72 + 17	73 = 72 + 1
161 = 144 + 17	145 = 144 + 1
233 = 216 + 17	217 = 216 + 1
305 = 288 + 17	289 = 288 + 1
377 = 360 + 17	361 = 360 + 1

This table summarises the basic structure of the pentagram presented in *Figure 4* and emphasises the central position of the 161 in this structure.

THE 161 AS CENTRE OF THE PENTAGRAM AND 23 JULY AS KEY QUANTITIES

Let us note the single and threefold occurrence of the 161 in the middle of the column of numbers as the actual centre of the pentagram, as *Figure 5* shows. If we break the 161 down into its prime factors, we obtain the numbers 23 and 07. In cultural history this combination of numbers was of significant importance, and so it was in ancient Egypt too, where on 23 July of the year the heliacal rising of Sirius could be observed, which indicated the rainy season and, connected with it, the period of the Earth's fertility. Among the Romans this day was the feast day of the water god Poseidon, primal father of leg-

endary Atlantis. In this context 23 July was always linked with the theme of (female) fertility and was of outstanding importance as a core aspect of creative action.

In the Christian tradition, too, the 23 and the 07 are causally connected with the "Redeemer", which plays a load-bearing role above all in the reckoning of time (beginning: 0 or 1 AD)
and in the dating of the historical events. In conclusion, I would still like to draw attention to the fact that a close astronomical context also exists between the 161 and the cycles of the Earth. Thus the duration of a Platonic year theoretically amounts to exactly $25920 + 1^2 = 25921$ years, which is based upon the square of the number 161, whereby, mathematically speaking, the following holds true: $161^2 = 25921$

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