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ESSAY · TRANSLATION

Proof and Truth

The Search for Proofs

German original: *Beweis und Wahrheit*

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THIS ESSAY IN OTHER LANGUAGES

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Kurt Gödel and his incompleteness theorem

PROVABILITY AND TRUTH

Kurt Gödel: "Über formal unentscheidbare Sätze der *principia mathematica* und verwandte Sätze" ("On Formally Undecidable Propositions of Principia Mathematica and Related Systems")

"All consistent axiomatic formulations of number theory contain undecidable statements."

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that is to say: there are statements that are *neither false nor correct, but unprovable*, yet true.

> "Do not read this sentence!" <

He showed that in a sufficiently rich axiom system that comprises the theory of numbers, there are always propositions that cannot be derived from the axioms, although the propositions are true. Consistency can only be proven if one passes over to a more comprehensive system. Gödel showed that *PROVABILITY* is a weaker concept than *TRUTH*, regardless of which axiomatic system is concerned.

The idea of making mathematics "introspective" was connected, in an extraordinarily fruitful way, with the notion of a code *("= Gödel numbering")*, in which numbers stand for symbols or sequences of symbols. Propositions of number theory can therefore be understood on two different levels:

* as a statement of number theory
* as a statement *about* statements of number theory

In this section I would like to go into the connection between science, proof and truth. The text above refers to the mathematician Kurt Gödel, who expressed himself on this in detail. His outstanding works prove, in principle, the open structure of living systems, which are linked with one another by meaning as if by a red thread and which are connected with one another through hierarchical functional attachments. They are coherently steered through resonances and, taken together, result in a unified and symbiotically aligned organism.

The following information on Gödel as a person and on the subject can be found on the internet as well as in countless books; I have shortened and condensed it to the essentials.

In 1931, in his main work "Über formal unentscheidbare Sätze der *principia mathematica* und verwandte Sätze" ("On Formally Undecidable Propositions of Principia Mathematica and Related Systems"), Gödel showed that in a sufficiently rich axiom system that comprises the theory of numbers, there are always propositions that cannot be derived from the axioms, although the propositions are true. Consistency can only be proven if one passes over to a more comprehensive system. According to Gödel's findings, mathematics is more than a formal system of axioms and rules of derivation.

In its absolutely purest version, Gödel's discovery represents the translation of an ancient philosophical paradox into the language of mathematics. It is the so-called *Epimenides* or *liar paradox:*

Epimenides was a Cretan who said: "All Cretans are liars"

Gödel's theorem: ***"All consistent axiomatic formulations of number theory contain undecidable statements."***

The idea behind it was the following: to use mathematical thinking for the exploration of mathematical thinking itself. This idea of making mathematics "introspective" was extraordinarily fruitful and led, among other things, to the incompleteness theorem. Gödel recognised that a statement of number theory can state something about a statement of number theory itself, and the core of his construction is the notion of a code *("= Gödel numbering")*, in which numbers stand for symbols or sequences of symbols. Every statement of number theory receives a Gödel number by which it can be designated. And this makes it possible to understand propositions of number theory on two different levels:

** as a statement of number theory*
** as a statement about statements of number theory*

Gödel's work forms only one part of the long-lasting efforts of mathematicians to become clear about what ***proofs*** are. It is important to note that proofs operate within fixed systems of statements. In Gödel's case, the fixed system of number-theoretical inferences to which the word proof refers is that of the *"principia mathematica" (p.m.)*, a gigantic work by *B. Russell* and *Alfred North Whitehead*, published from 1910 to 1913.

In colloquial language, Gödel's theorem means: For this proposition of number theory there are no proofs whatsoever in the system of the p.m.
That is to say: the paradox is *neither false nor correct, but unprovable*, yet true. From this it follows that there are true statements of number theory for whose proof the methods of the p.m. are too weak. Gödel showed that *provability* is a weaker concept than *truth*, regardless of which axiomatic system is concerned.

In the final consequence it follows from this that "provability" and "truth" stand antagonistically opposed to each other. If one deals with "absolute truth", there is no possibility of a proof, and if one moves on the level of provability, a part of the truth is always missing.

PROOF AND TRUTH

Example

> *A person stands at the edge of the forest and observes a shy deer that no one has ever seen before. It remains visible for a short time and then disappears into the forest again.*

For the person concerned the experience is genuine and true, for society, however, it remains a myth (superposition) until it has also been observed by other people. Since no one can know whether the animal in question will reappear (or will remain in the forest forever, whether it perhaps dies immediately afterwards and therefore can never become visible again), provability remains a riddle.

From this example we recognise that the process of furnishing proof is a weaker argument than the truth (actuality) of being (see K.Gödel). In the act of proving, we replace the actuality of the "it is" with a partial copy that builds on comprehensible similarity and order. If the actuality of the perception of an object is highly illogical, society denies this event the predicate true, regardless of the real event. If no one (or no expert) can repeat the same experiences (experiments), the fact remains settled in the realm of fantasy. It is evident that the furnishing of proof can never judge the truth or actuality of an individual and subjective event of perception, but has as its inner core a restricted definition of the "it-is state" in the form of a collectively accepted world view.
The "proof" as such is therefore, despite its misleading definition, a process of socialising actuality, in which the aspects of the whole truth accessible to an elite (experts) are laid down. Thus every proof of this category is completely unsuited to grasp or judge individual truths, because they do not render primary actualities of being, but define a process of approaching these individually graspable subjective truths.

Furthermore, the whole can never be reconstructed on the basis of a partial aspect, which, in the case of the form of furnishing proof described here, is part of it. Collective access to truth runs through mediators (experts, priests, shamans etc.), who span the bridge from possibility to actuality, a process that depends on the available evidence or so-called facts. These in turn are bound to the respective state of knowledge of a society, which represents a radical restriction of the possibilities of perception of a culture. What is not generally accepted cannot be found elsewhere either, for in order to accept a fact or truth, one needs an ordered system that makes the embedding of the new possible.

NEGATION AND THE FUNCTION OF THE PROOF MIRROR

Proof, in its ideal-typical positive understanding, does not depict reality, but mirrors and copies it as negation. Just as a mirror, as a permanent state of the original emptiness, negates every form of bodily reality, so proof is the negative of truth as it exists in the unique moment. The mirror negates the world because it reflects everything and refuses to take anything object-like into itself. Thus no image of the world arises in its interior, which leaves out bodily reality.
Proof – which contains repeatability and comprehensible logic as central elements – in this sense negates and copies truth through its "not-being-so" or its opposite. Where truth still builds on spon-

taneity and chaos, in proof we find repeatability and comprehensible logic, aspects that are missing in the core of the concept of truth. Furthermore, proof mirrors truth in an endless fractal chain, and like every mirror image, the mirror reflects the substance without containing it. Proof thus illustrates the path that subjective truth travels until its integration into the body of society. A path, however, can never be used for testing a fact and a snapshot, which is why a "proof" as such is a contradiction in itself.

If we follow the ideal-typical logic of reference and interpret truth as a singular and subjectively experienceable snapshot of a state of being, then proof mirrors this moment decimally deformed by one dimension. This chain of proof can be extended at will and accumulates in a more comprehensive truth than exists at the beginning. The collective contains all mirror images of the same original motif/object as a "sum", whereby the primary truth is contained, expanded, in the limit of the final result.

e.g.: original image = truth = archetype number 18

As the first decimal deformation (first mirror) we find the number 1.8, as the second decimal deformation (second fractal mirror) the number 0.18 and so on. The sum results from:

$$\Sigma\ 18\ +1.8\ +0.18\ +\ 0.018\ +\ldots\ +\ =\ \Sigma\ 20$$

In this process the original truth is always preserved, albeit distorted, and with the sum and the number 20 it provides an expansion that goes beyond the individual self-reflective images.

If the repeated perception of the original truth is almost identical with it, then the chain of proof contains no additional information whatsoever with regard to the primary truth. In this case the snapshot is fractally mirrored but not expanded, which makes the proof superfluous, because no new information is added.

If the ongoing furnishing of proof contains distorted information with regard to the primary truth, then the fractal mirror image is clearly not identical with the truth and therefore likewise not suitable as proof. Thus we recognise the immanent paradox of finding truth through a proof: the power of proof lies in the greatest possible negation of truth.

If proof and truth are the same, the subjective truth determines the output of the system, for if the "truth" is true, so is the proof, and nothing more. If the original perception is an illusion, the proof too can only render this illusion and is therefore superfluous. If we assume that the proof deviates more or less from the truth, the simple conclusion results that two different things are present, in consequence of which the proof cannot depict the truth.

The depiction of truth through proof therefore takes place as a *threefold negative* mirroring:

Truth	Proof
chaos, irrationality	comprehensible logic
singularity	repeatability
snapshot	process

The chain of evidence of proof ideal-typically forms the shell aspect of truth, which, on the level of consciousness, as the cardinal part, embodies the content or nectar that is embedded in the vessel of proof.

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